

MST124

TMA 03

2018J

Covers Units 7 and 8

Cut-off date 13 March 2019

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment' area of the MST124 website.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 11. You do not have to submit any work for Question 11.

Question 1 – 10 marks

You should be able to answer this question after studying Unit 7.

Differentiate the following functions.

(a) $f(x) = \ln(3x)e^{2x}$. [3]

(b) $g(x) = \frac{x^2 - 1}{e^{-4x} - 3}$. [3]

(c) $h(t) = \ln(t^3 + 2t^2)$. [4]

Question 2 – 5 marks

You should be able to answer this question after studying Unit 7.

Differentiate the function

$$k(x) = \frac{\sin x}{(x^4 + 3x)^5}. \quad [5]$$

Question 3 – 10 marks

You should be able to answer this question after studying Unit 7.

A piece of wire of length 30 cm is cut into two parts. The first part is bent into a circle of radius r (in cm), the second part into a rectangle that has sides of lengths x (in cm) and $2x$ (in cm).

(a) Write down an expression for the sum of the perimeters of the two shapes in terms of r and x . Use this to express r in terms of x . [3]

(b) Find an expression for S , the sum of the areas enclosed by the two shapes, in terms of x . [2]

(c) Use differentiation to determine the value of x for which S takes a minimum value. Give your solution to one decimal place. [5]

Question 4 – 5 marks

You should be able to answer this question after studying Unit 7.

Find the indefinite integral of the function

$$f(x) = \frac{x^2 e^x + 3}{x^2}. \quad [5]$$

Question 5 – 10 marks

You should be able to answer this question after studying Unit 8.

Use integration by substitution to find the indefinite integral in part (a) and to evaluate, to two decimal places, the definite integral in part (b).

(a) $\int \frac{\sin x}{(3 \cos x - 4)^5} dx$ [4]

(b) $\int_0^1 x^3 e^{3x^4} dx$ [6]

Question 6 – 5 marks

You should be able to answer this question after studying Unit 8.

Use integration by parts to find the indefinite integral

$$\int (2x - 1) e^{-x} \, dx. \quad [5]$$

Question 7 – 10 marks

You should be able to answer this question after studying Unit 8.

This question is about the function

$$f(x) = x^4 \ln x.$$

- (a) Explain why the graph of f lies on or above the x -axis for all values of x greater than or equal to 1. [3]
- (b) Write down an expression, involving a definite integral, that gives the area between the graph of f and the x -axis, from $x = 1$ to $x = 5$. [1]
- (c) Use integration by parts to find the area described in part (b), evaluated to two decimal places. [6]

Question 8 – 10 marks

You should be able to answer this question after studying Unit 8.

- (a) Use integration by substitution to find the indefinite integral

$$\int (x^2 + 3) (x - 1)^7 \, dx. \quad [5]$$

- (b) Use integration by parts to find the indefinite integral

$$\int x^2 \sin(3x) \, dx. \quad [5]$$

Question 9 – 10 marks

You should be able to answer this question after studying Unit 8.

Include a printout or screenshot of your Maxima worksheet for this question.

You are not expected to annotate your Maxima worksheet with explanation.

However, remember that for good mathematical communication you should present your answers clearly.

This question is about the function

$$f(x) = \frac{-6(x^6 + 6x^3 - 3x)}{x^3 + 1}.$$

Use Maxima to do each of parts (a)–(d).

(In parts (c) and (d) you should round your answers to three decimal places yourself, instead of using Maxima to do this.)

- (a) Plot the graph of f , choosing ranges of values on the x - and y -axes to make its stationary points clearly visible. [3]
- (b) Find the derivative of f . [1]
- (c) Calculate the x - and y -coordinates of the local maximum of f , to three decimal places. [3]
- (d) The graph of f crosses the x -axis at $x = 0$. Calculate the value of x where the graph crosses the x -axis to the right of $x = 0$. Find the area enclosed by the graph of f and the x -axis between $x = 0$ and this value, giving your answer to three decimal places. [3]

Question 10 – 20 marks

You should be able to answer this question after studying Units 1 to 8.

In this question, you are asked to attempt the mini examination paper on pages 7–9 and enter your answers on an answer form of the type that you will use in the real examination (see page 6). This will give you practice in completing the form.

You must submit your completed answer form as part of this TMA. If you are posting the TMA, then print and complete the form, and include it with your TMA. If you are submitting electronically, then either print and complete the form and scan or photograph it to produce a PDF, or complete it electronically by using annotation software. In either case you must then incorporate it into your TMA. There is a copy of the form as a single-page PDF in the ‘Assessment’ area of the MST124 website.

You are not required to submit any working for the mini examination paper.

Of the 20 marks for this question, 18 marks are for correct answers to the questions in the mini examination paper, and 2 marks are for completing the form correctly.

Print or download a copy of the computer-marked examination form on page 6, read the instructions for completing it on page 7, and complete Part 1 of the form. An example of a completed form can be found in the Examination and revision resources section of the website at <https://learn2.open.ac.uk/course/view.php?id=207065&cmid=1306239>

Then work through the mini examination paper on pages 7–9, and mark your answers on Part 2 of the form.

[20]

Question 11 – 5 marks

Your score out of 5 marks for good mathematical communication in Questions 1 to 9 will be recorded under Question 11.

You do not need to submit any work for this question.

[5]

COMPUTER MARKED EXAMINATION FORM

08-10

PART 1

PERSONAL IDENTIFIER
(NOT Examination Number)
ASSIGNMENT NUMBER
(as given on the question paper)

NAME and INITIALS

IMPORTANT NOTES

1. Please check that all of this form has been completed including the coloured boxes in Part 1 with your Personal Identifier, Module and Assignment Number.
2. Only use an HB pencil to complete this form and press firmly.
3. Shown opposite are examples of the correct marks to use. Do not use any other types of mark.

PLEASE DO NOT PUNCH HOLES
IN THIS FORM

Fold this form INWARDS along the dotted line

PERSONAL IDENTIFIER

A	0	0	0	0	0	0	0
B	1	1	1	1	1	1	1
C	2	2	2	2	2	2	2
D	3	3	3	3	3	3	3
E	4	4	4	4	4	4	4
F	5	5	5	5	5	5	5
G	6	6	6	6	6	6	6
H	7	7	7	7	7	7	7
K	8	8	8	8	8	8	8
L	9	9	9	9	9	9	9
M							X
N							
P							
R							
S							
T							
U							
V							
W							
X							
Y							

CORRECT MARK

Pencil across
between dots

CANCELLATION MARK

To cancel a mark
pencil in the
coloured part of
the cell. Do not rub out

MODULE AND ASSIGNMENT NUMBER

A	A	A	A	0	0	0	0
B	B	B	B	1	1	1	1
C	C	C	C	2	2	2	2
D	D	D	D	3	3	3	3
E	E	E	E	4	4	4	4
F	F	F	F	5	5	5	5
G	G	G	G	6	6	6	6
H	H	H	H	7	7	7	7
K	K	K	K	8	8	8	8
L	L	L	L	9	9	9	9
M	M	M	M				
N	N	N	N				
P	P	P	P				
R	R	R	R				
S	S	S	S				
T	T	T	T				
U	U	U	U				
W	W	W	W				
X	X	X	X				
Y	Y	Y	Y				
Z	Z	Z	Z				

OFFICE USE ONLY

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

OFFICE USE ONLY



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PART 2

ANSWER

ANSWER

ANSWER

1	A	B	C	D	E	F	G	H	?	U	21	A	B	C	D	E	F	G	H	?	U	41	A	B	C	D	E	F	G	H	?	U
2	A	B	C	D	E	F	G	H	?	U	22	A	B	C	D	E	F	G	H	?	U	42	A	B	C	D	E	F	G	H	?	U
3	A	B	C	D	E	F	G	H	?	U	23	A	B	C	D	E	F	G	H	?	U	43	A	B	C	D	E	F	G	H	?	U
4	A	B	C	D	E	F	G	H	?	U	24	A	B	C	D	E	F	G	H	?	U	44	A	B	C	D	E	F	G	H	?	U
5	A	B	C	D	E	F	G	H	?	U	25	A	B	C	D	E	F	G	H	?	U	45	A	B	C	D	E	F	G	H	?	U
6	A	B	C	D	E	F	G	H	?	U	26	A	B	C	D	E	F	G	H	?	U	46	A	B	C	D	E	F	G	H	?	U
7	A	B	C	D	E	F	G	H	?	U	27	A	B	C	D	E	F	G	H	?	U	47	A	B	C	D	E	F	G	H	?	U
8	A	B	C	D	E	F	G	H	?	U	28	A	B	C	D	E	F	G	H	?	U	48	A	B	C	D	E	F	G	H	?	U
9	A	B	C	D	E	F	G	H	?	U	29	A	B	C	D	E	F	G	H	?	U	49	A	B	C	D	E	F	G	H	?	U
10	A	B	C	D	E	F	G	H	?	U	30	A	B	C	D	E	F	G	H	?	U	50	A	B	C	D	E	F	G	H	?	U
11	A	B	C	D	E	F	G	H	?	U	31	A	B	C	D	E	F	G	H	?	U	51	A	B	C	D	E	F	G	H	?	U
12	A	B	C	D	E	F	G	H	?	U	32	A	B	C	D	E	F	G	H	?	U	52	A	B	C	D	E	F	G	H	?	U
13	A	B	C	D	E	F	G	H	?	U	33	A	B	C	D	E	F	G	H	?	U	53	A	B	C	D	E	F	G	H	?	U
14	A	B	C	D	E	F	G	H	?	U	34	A	B	C	D	E	F	G	H	?	U	54	A	B	C	D	E	F	G	H	?	U
15	A	B	C	D	E	F	G	H	?	U	35	A	B	C	D	E	F	G	H	?	U	55	A	B	C	D	E	F	G	H	?	U
16	A	B	C	D	E	F	G	H	?	U	36	A	B	C	D	E	F	G	H	?	U	56	A	B	C	D	E	F	G	H	?	U
17	A	B	C	D	E	F	G	H	?	U	37	A	B	C	D	E	F	G	H	?	U	57	A	B	C	D	E	F	G	H	?	U
18	A	B	C	D	E	F	G	H	?	U	38	A	B	C	D	E	F	G	H	?	U	58	A	B	C	D	E	F	G	H	?	U
19	A	B	C	D	E	F	G	H	?	U	39	A	B	C	D	E	F	G	H	?	U	59	A	B	C	D	E	F	G	H	?	U
20	A	B	C	D	E	F	G	H	?	U	40	A	B	C	D	E	F	G	H	?	U	60	A	B	C	D	E	F	G	H	?	U

MST124 Mini examination paper

This paper has **TWO** sections. You should attempt **ALL** questions in each section.

Section A has 5 questions, each worth 2 marks.

Section B has 2 question, worth 4 marks.

Each question is multiple-choice, with **ONE** correct answer from five options. No marks will be deducted for incorrectly answered questions.

Mark your answers on the computer-marked examination (CME) form provided. Instructions for filling in the CME form are given below.

The instructions below are almost identical to those that will be provided in the real examination. You should follow the instructions carefully, just as you would in the real examination, except that the instruction to use an HB pencil is irrelevant if you are annotating the form electronically for this TMA, and of course the instruction that if you need a new form then you should ask an invigilator does not apply for this TMA.

Instructions for completing the CME form

If you do not follow these instructions, then the examiners may not be able to award you a score for the examination.

Writing on the form

- Use an HB pencil.
- To mark a cell, pencil across it, as demonstrated on the form.
- To cancel a mark, pencil in the coloured part of the cell, as demonstrated on the form.
- If you make any unwanted marks on the form that you cannot cancel clearly, then ask the invigilator for a new form, and transfer your entries to it.

Completing Part 1

- Enter your personal identifier (NOT your examination number) and the ‘assignment number’ for this examination, which is **MST124 81**, in the boxes provided.
- In the blocks headed ‘PERSONAL IDENTIFIER’ and ‘MODULE AND ASSIGNMENT NUMBER’, pencil across the cells corresponding to your personal identifier and the assignment number given above.

Completing Part 2

- For each question (numbered 1 to 7), mark your answer by pencilling across ONE of cells A, B, C, D or E.
- If you think that a question is unsound in any way, pencil across the ‘unsound’ cell (‘U’), *as well as* pencilling across an answer cell.
- Do not pencil across any other cell.

SECTION A

Question 1

For a certain function g , it is known that $\int_{-3}^5 g(x) \, dx = -2$,
 $\int_{-3}^2 g(x) \, dx = -3$ and $\int_4^5 g(x) \, dx = -1$.

What is the value of $\int_2^4 g(x) \, dx$?

- A** -6 **B** 2 **C** 6 **D** -2 **E** 0

Question 2

Given that

$$y = \sqrt{x}e^{2x},$$

which of the following corresponds to $\frac{dy}{dx}$?

- A** $\frac{1}{2}x^{-1/2}e^{2x} + x^{1/2}e^{2x}$ **B** $2x^{-1/2}e^{2x} + 2x^{1/2}e^{2x}$ **C** $x^{-1/2}e^{2x}$
D $-\frac{1}{2}x^{-1/2}e^{2x} + 2x^{1/2}e^{2x}$ **E** $\frac{1}{2}x^{-1/2}e^{2x} + 2x^{1/2}e^{2x}$

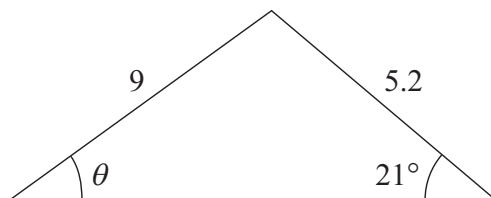
Question 3

Which of the following is equivalent to $(x + 3)(2x + 1) + 4x(x - 3)$?

- A** $6x^2 - 12x + 3$ **B** $6x^2 + 19x + 3$ **C** $8x + 1$
D $2x^2 + 11x - 9$ **E** $6x^2 - 5x + 3$

Question 4

The triangle below (which is not drawn to scale) has two sides of lengths 9 and 5.2. The angle opposite the side of length 9 is 21°



What is the size of the angle θ , to two decimal places, opposite the side of length 5.2?

- A** 11.94° **B** 11.95° **C** 0.48° **D** 38.33° **E** 38.34°

Question 5

What is the rule of the inverse function of $f(x) = 6x - 1$?

- A** $f^{-1}(x) = \frac{1}{6x-1}$ **B** $f^{-1}(x) = \frac{6-x}{6}$ **C** $f^{-1}(x) = 6x - 6$
D $f^{-1}(x) = \frac{x+1}{6}$ **E** $f^{-1}(x) = \frac{6x+1}{6}$

SECTION B**Question 6**

Find, to the nearest degree, the angle between the vectors $2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{i} + 4\mathbf{j}$.

- A** 144° **B** 3° **C** -54° **D** 36° **E** 54°

Question 7

An object starts from rest at time $t = 0$ and moves along a straight line with velocity v in ms^{-1} , given by $v = 3t(2 - t^2)$.

Which of the following gives the distance in metres travelled by the object when it next comes to rest?

- A** 9 **B** -12 **C** 3 **D** 0 **E** 12

[END OF QUESTION PAPER]
